22087216

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Monday 19 May 2008 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 10]*

(a) Find the value of $\lim_{x\to 1}\left(\frac{\ln x}{\sin 2\pi x}\right)$. *[3 marks]*

(b) By using the series expansions for e^{x^2} and $\cos x$ evaluate $\lim_{x\to 0}\left(\frac{1-e^{x^2}}{1-\cos x}\right)$. *[7 marks]*

2. *[Maximum mark: 9]*

Find the exact value of $\int_0^\infty \frac{dx}{(x+2)(2x+1)}$.

3. *[Maximum mark: 14]*

A curve that passes through the point $(1, 2)$ is defined by the differential equation

$$\frac{dy}{dx} = 2x(1+x^2-y) .$$

(a) (i) Use Euler's method to get an approximate value of y when $x = 1.3$, taking steps of 0.1. Show intermediate steps to four decimal places in a table.

(ii) How can a more accurate answer be obtained using Euler's method? *[5 marks]*

(b) Solve the differential equation giving your answer in the form $y = f(x)$. *[9 marks]*

4. *[Maximum mark: 14]*

(a) Given that $y = \ln\cos x$, show that the first two non-zero terms of the Maclaurin series for y are $-\frac{x^2}{2}-\frac{x^4}{12}$. *[8 marks]*

(b) Use this series to find an approximation in terms of π for $\ln 2$. *[6 marks]*

5. *[Maximum mark: 13]*

(a) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)\,3^n}$. *[6 marks]*

(b) Determine whether the series $\sum_{n=0}^{\infty} \left(\sqrt[3]{n^3+1} - n\right)$ is convergent or divergent. *[7 marks]*
